

PERFORMANCE TASK SAMPLE

Math

Algebra I

World Domination

Should we worry? The US currently has the highest GDP in the world, but what will the world look like in 2050, if current trends continue? Young economists prepare a presentation to the World Economic Forum in Davos.

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World Domination

Principal math competency assessed:

CONSTRUCT SOUND ARGUMENTS

Draw and defend conclusions about what data mean, and develop sound conclusions and proofs, based in logical thinking and appropriate evidence.

Additional math competencies assessed:

INVESTIGATE

Pose or address a non-routine mathematical problem, formulate a problem-solving approach, and gather the information necessary to solve the problem.

DEVELOP INTERPRETATIONS & EXPLANATIONS

Develop mathematical models by which phenomena and data can be better understood and made to yield powerful predictions and generalizations.

WORK STRATEGICALLY

Make and act on deliberate choices about what to do, when and how to do it, and how best to negotiate challenges along the way.

COMMUNICATE

Communicate historical and economic information and ideas clearly and effectively, adjusting to address different audiences and purposes.

CONTRIBUTE

Work effectively as an individual, a member of a team, and a member of society to build communities that support a shared commitment to excellence in inquiry, analysis, interpretation, and communication; and to build a better society.

Common Core Standards addressed and assessed at a major level:

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
8. Look for and express regularity in repeated reasoning.

High School Content Standards

Algebra

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Functions

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
 - a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
 - a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
 - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Statistics and Probability

Interpret linear models

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.



World Domination

Student assignment sheet

Background:

"Gross domestic product," or GDP, is the measure of the total value of goods and services sold in a country in a given year. Economists often use this number to judge the health of a country's economy: to a great extent, the higher the GDP, the better the economy. Below is a chart that shows the GDP of five countries in 2000 and 2010.

Gross Domestic Product, in billions of US dollars, in 2000 and 2010

Country	2000 GDP	2010 GDP
United States	9825	13,271
European Union	9395	12,965
Japan	4176	4,601
China	1078	2998
India	469	929

Source: Goldman Sachs, via www.geographic.org

But this chart only tells us what the GDP was in the past. What will the future look like? If current trends continue, who will be the economic powerhouse in the year 2050? In the year 2075?

Your task:

You are an economist. Your team has been hired by the World Economic Forum to investigate trends in GDP in countries that are economically powerful today or are expected to become economically powerful in the future: the US, the European Union, Japan, China, and India.

Using the data in the table above, and assuming that the rate of growth between 2000 and 2010 will remain constant for the next 75 years, find the GDP for the years 2050 and 2075. Because you will be presenting your work to a broad group of people, you will include a verbal description (in writing or orally) that also includes algebraic equations and graphs. Your job is to answer the question, "Which will be the dominant economy in the future?" You may present your final report:

- in a written position paper
- as a videotaped presentation
- as a web page
- in another format that respects the seriousness of this work

***IMPORTANT NOTE:** Change in GDP is not necessarily linear, and current patterns might not continue. When you prepare your final report, be sure to clearly state that your projections are based on the assumption that current rates of growth will remain constant over the period of time that your projections cover.

World Domination

Teaching Notes

These notes contain a high level of detail, in an effort to clarify one way in which this performance task might unfold. Along with guiding the assessment itself, they integrate learning activities that prepare students for the assessment task. Teachers are welcome to either follow these steps or create their own structures for teaching and assessment.

Part 1, a learning activity:

Students explore measures of national economies.

- Assess prior knowledge and gauge economic thinking by asking these questions:
 - Is the US a wealthy country? How do you know?
 - What are some other wealthy countries? How do we know?
 - What kinds of mathematical measures might we make to determine whether a country is wealthy?
 - How might we tell whether a country is increasing or decreasing its wealth?
- Introduce the concept of gross domestic product and help students understand how it is developed. Consider the attached graphs of various aspects of GDP in different countries. What conclusions can they draw by comparing the charts?

Part 2, strategies for problem solving

Students work in groups to develop strategies for solving the problem.

- Introduce the performance task and divide students into working groups. Discuss the rubric, and allow time for questions.
- Allow each team time to develop a preliminary strategy for completing the performance. First, have the group work together to explain what they will need to do in order to answer the questions about GDP for 2050 and 2075. (Teachers should monitor the group conversations to see if students are on track.)
- Meet with each group, asking it to explain its strategy. Ask questions about the chosen approach, asking about anticipated challenges, distributions of the work load, and structures for communication as the work proceeds.

Part 3, the task itself:

Students consider the different perspectives revealed through different texts.

- Groups work as independently as possible to complete the task.
- Groups present their findings to the World Economic Forum at Davos.

Part 4, learning from the assessment:

Students explore questions of confidence in their solutions, strategic challenges they faced, and successes and challenges of collaboration.

- Help students connect their predictions with real life. If their predictions are accurate, how might the US prepare for the future? Foster a short discussion in which students build upon each others' thinking to consider changing patterns of consumption, altering the composition of GDP, preparing for the possibility that the US will not remain the chief economic power, and/or embracing a future in which the US is not the chief economic power.
- Ask students how confident they are with their predictions, asking them to explain and justify their answers. Recall that we are assuming for the task that the rate of GDP growth is constant. Though this is not an economics class, ask students to discuss whether they think that is a good assumption. Go here, for example, for US GDP rates per year since 1930, from the US Dept. of Commerce in an Excel spreadsheet: <http://www.bea.gov/national/index.htm#gdp>
- Here is international data:
[http://en.wikipedia.org/wiki/List_of_countries_by_real_GDP_growth_rate_\(latest_year\)](http://en.wikipedia.org/wiki/List_of_countries_by_real_GDP_growth_rate_(latest_year))
- On China: <http://www.chinability.com/GDP.htm>
- How does this real-world year-to-year variation affect their confidence in their predictions?
- Have students write a short reflection on these two “process” questions:
 1. At the beginning of this task, your group developed a strategy for solving the problem. Did the strategy work perfectly for you? If not, how did you adjust?
 2. How effective were you as a collaborative team member? What three things might you do during your next group assignment to be a better team member?

Possible Learning Activities:

These activities might form the heart of the Stage 3 learning plan, to help students build background knowledge and skill, deepen their understanding of effective arguments, and practice the kind of writing and/or speaking that they will require for their assessment. Some of these learning activities may be completed in earlier units, others within this unit. The activities listed here are only samples, and this is not intended to map a comprehensive learning plan for the unit.

- Direct instruction and practice in key mathematical content, including:
 - develop equations with single and multiple variables
 - graph linear equations
 - interpreting graphs
 - determining the intersections of lines on a graph, using algebra

For example:

Essential Questions:

- ❑ What are the defining characteristics of a type of relationship? How do we best measure and interpret them?
When would I prefer one algebraic representation of a line over another?
Is there always only one best option?
How do we best measure and interpret the unique characteristics of linear relationships?

Characteristics of Lines

Students will explore using slope and y-intercept as defining characteristics of lines.

1. Students will need graphing calculators, or a computer graphing program, to explore a larger number of linear equations and make observations regarding slope and y-intercept.
2. Give students a set of simple equations in y-intercept form with positive slopes ranging from 1 to 5; let the y-intercept = 0. Ask them to make observations as to the effect of the multiplier. Don't use the word slope yet. If a student uses the term, ask her to re-word what they want to say without it. Repeat step 2 using slopes between 0 and 1. Repeat step 2 using slopes between -1 and -5. Repeat step 2 using slopes between 0 and -1. Ask students to summarize how this multiplier affects the graph of the line. Don't use the word "slope." Use language like, "rises to from left to right," "falls from left to right," "is steep," "is not steep." - OR whatever language the class came up with when first describing non-horizontal points.
2. Have students graph a set of equations with the same slope, but different y-intercepts ranging from -5 to 5. Have them describe the effect of adding a constant. Do not use the term y-intercept yet. If a student uses the term, ask her to re-word what they want to say without it.
3. Give the students a set of equations with varied slopes and y-intercepts. Ask them to write down what they expect to see when they graph these lines. Have them use the graphing calculator to check their guesses.

Graphing Linear Equations

Students will receive direct instruction in graphing linear equations.

1. Use the previous activity to introduce slope-intercept form.
2. Discuss slope in various ways including: real-world examples with units (ex: mph), as it describes a characteristic of the line, algebraically.
3. Give students a line with multiple points labeled and have them generate the slope from various pairs of points on the line to demonstrate the idea of constant slope. What might a shape look like that does not have constant slope?
4. Give students a point and a slope and ask them to graph all the lines that could have that point and slope (only one line). If that information determines exactly one line, it must determine exactly one equation. How can we find that equation?

This is a good time to reinforce Polya's problem-solving approach - U.P.E.R.: UNDERSTAND the problem, create a PLAN, EXECUTE the plan, and REVIEW the answer. In this case, we already know how to write the equation of a line if we know the y-intercept and the slope. If we have slope and a point on the line, we need a PLAN to figure out the y-intercept.

Give students two points and ask them to graph them. How many lines go through those two points? (Only one.) If that information determines exactly one line, it must determine exactly one equation. How can we find that equation?

Again, consider the problem-solving approach - this time we don't know either the slope or the y-intercept. We need a plan to use the information that we know to find the slope and the y-intercept.

6. In the interest of less memorizing and more understanding, always refer back to slope-intercept form when writing equations. For instance, if you know two points, you can generate slope, and then plug one of the points and the slope into " $y = mx + b$ " and solve for "b."

The Meaning of 'Linear' and its Implications

Ask students to propose and verify two different real-world relationships that are linear in a topic of interest to them. Now, ask them to predict what will happen when each of the two elements varies, in terms of equation/slope – and what such variations means in the real world. e.g. what happens to time when distance stays the same but speed has increased?

Then, have them identify a real-world relationship which is not linear using the same quantities (e.g., how velocity varies over time). How might they now describe in natural language the difference between linear and non-linear relationships? (What's the difference between a graph of line segments of varying slope and a non-linear curve where the slope is never constant?) What would they predict as to population growth over time – linear or non-linear? What about population to automobiles? What about people to cows? Farmers to cows? # of Students to # of students sick on any given day?

- Discussion of what a mathematical argument is and how to make an effective one, including consideration of the value of verbal, algebraic, and graphic models.
- Analysis of sample mathematical arguments, including arguments that use math to address real-world situations. Sample arguments should represent a range of quality, from tight and complete through sloppy or illogical.
- Opportunities to work collaboratively to solve shorter problems, with active evaluation of the quality of that collaboration.
- (Also, the learning activities detailed in the “Teacher Notes” to the performance task.)

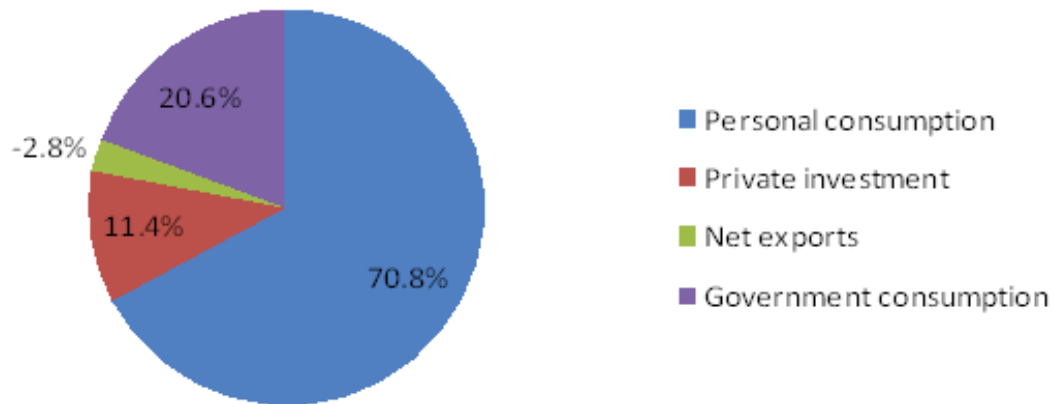
Ideas for Differentiation

(the beginnings of a list)

- help students find roles within the groups that both challenge them appropriately and allow them to build upon their strengths
- extend the task for accomplished mathematicians by asking them to identify the year in which China will surpass the US in GDP, defending their reasoning algebraically
- provide additional instruction or practice sets for students struggling with the basic mathematics

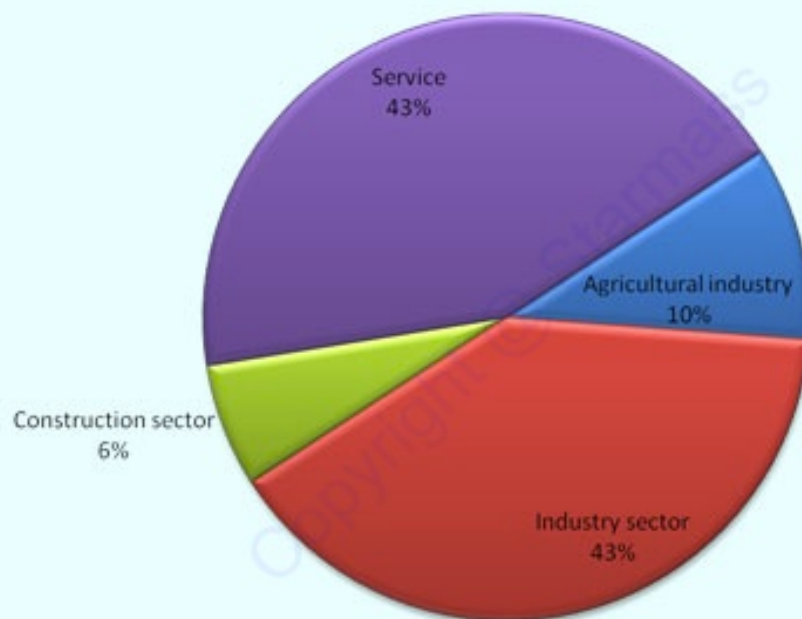
GDP CHARTS

US GDP by type of income 2009



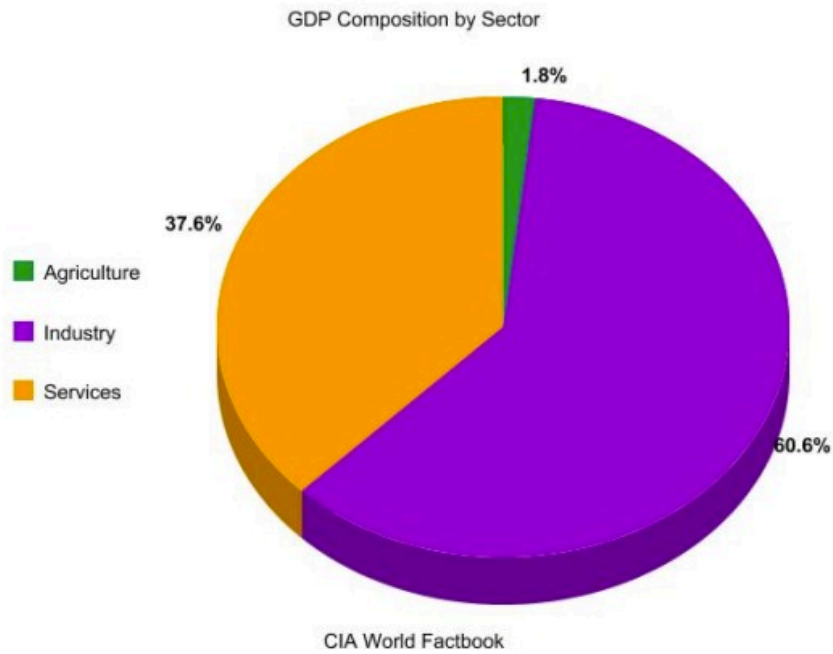
Source: www.sociologysystemsresearch.com

Below is the chart of China GDP by industry for the year of 2009:



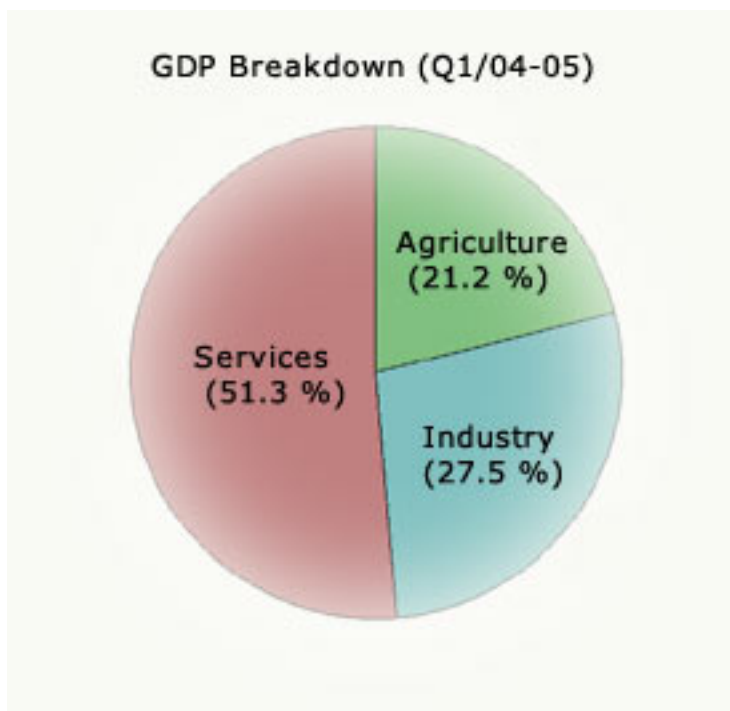
Source: www.starmass.com

GDP Composition By Sector Graph.jpg



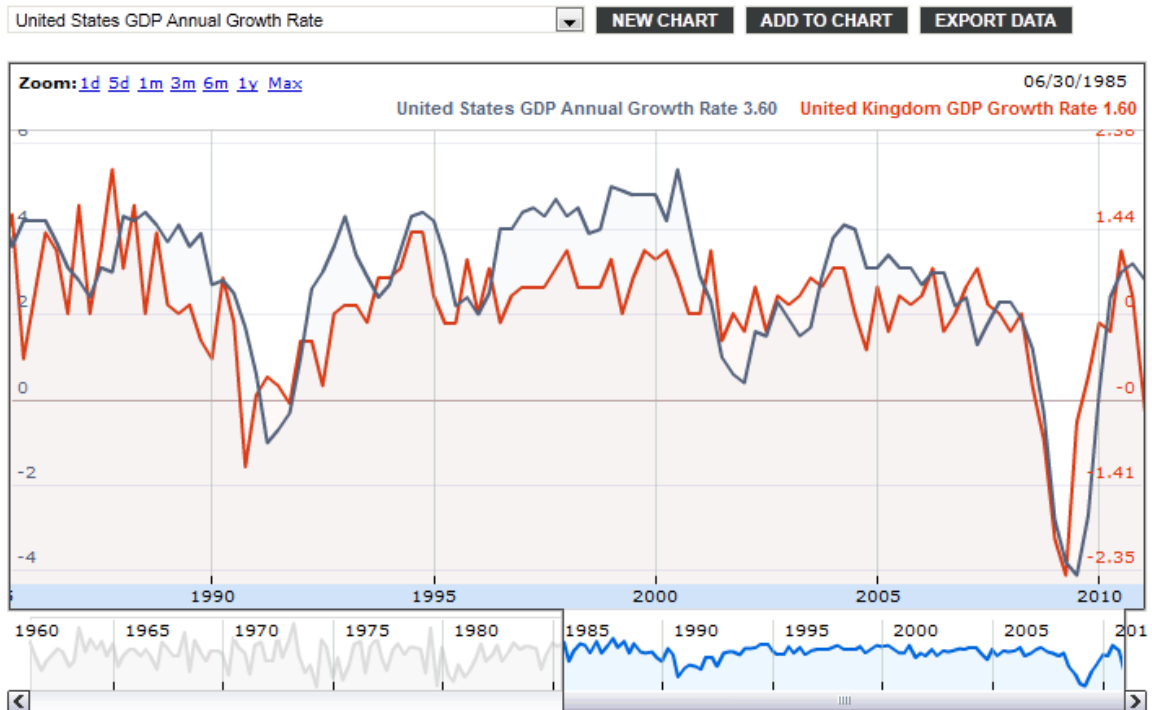
Source: internationalbusiness.wikia.com

India GDP Components



Source: www.indiabuzzing.com

GDP – actual trends



GDP accumulated growth, in percent, constant prices

